Acta Technica *61*, No. 4B/2016, 87–98

Mathematical model to predict production of fractured horizontal well in tight gas reservoir and production analysis[1]

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Abstract. According to different flow state around the fractured horizontal well associated with the threshold pressure gradient (TPG), combined with mass and momentum conservation equations, a mathematical model for steady gas flow in single fracture of fractured horizontal well was established. Considering the effect of interference between the fractures during the production period, production prediction model of fractured horizontal well was presented. Numerical analysis shows that the longer the fracture half-length, the bigger the productivity, but the productivity growth is reduced, and there is an optimal fracture half-length; the bigger the fracture conductivity, the bigger the productivity; the more the fracture quantity, the bigger productivity, but productivity growth is reduced due to the interference between fractures.

Key words. Tight gas reservoir, fractured horizontal well, fracture disturbance, elliptical flow.

1. Introduction

Tight sandstone gas resources in the global energy mix more and more important, is a strategic area at oil and gas exploration and development in future. In recent years, abundant tight sandstone gas resources were discovered in Ordos, Sichuan, Turpan-Hami basins, and its reserves, natural gas production accounts for one-third in China [1–4]. Compare to conventional gas reservoirs, it is difficult to develop

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[1] This work is supported by National Program on Key Basic Research Project (973 Program) (Grant No. 2013CB228002), National Natural Science Foundation of China (Grant No. 11372033), and Fundamental Research Funds for Central Universities (No. FRF-TP-15-043A1).

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and achieve economic production less than expected using only horizontal wells due to complex geological occurrence of tight gas reservoir, dense reservoir, and small effective discharge area around the wells [5–6]. Therefore, multi-stage fractured horizontal wells are used to improve the well production and achieve cost-effective development. Numerical Simulation method is often used to study on production of fractured horizontal well, but complex data sources, long evaluation period, the application is not simple and easy [7–9]. Combined mass and momentum conservation equations, considering different flow characteristics [10–13], the fluid flow around the fracture is divided into two regions, a mathematical model of fractured horizontal wells in tight gas reservoirs was established, the analytic solution was obtained, productivity prediction equation was derived to evaluate development effect of fractured horizontal well intuitively.

2. Flow characteristics

The phenomena of non-Darcy flow in tight gas reservoir (TPG, slippage effect, and high-velocity non-Darcy flow) can be theoretically explained from the molecular force.

There are three forces during gas flow in the core: molecular adsorption between gas molecules and rock surface, interaction between gas molecules and differential pressure on both ends of cores.

Figure 1 shows the relationship between permeability and pressure gradient.

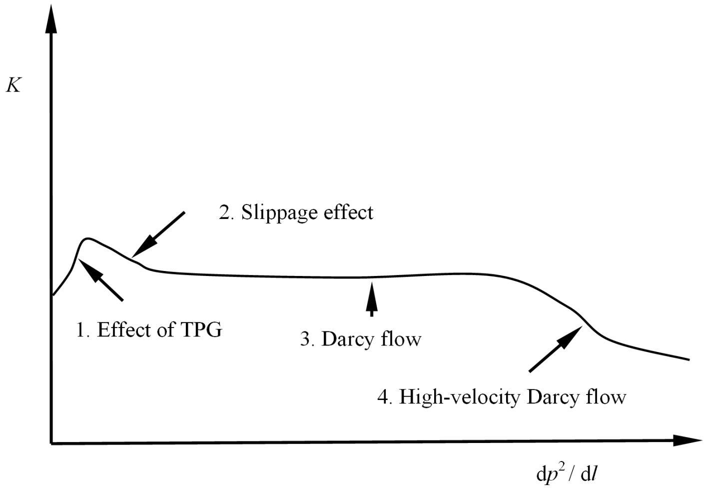

Fig. 1. Relationship between permeability and pressure gradient

As seen in the first stage of Fig. 1, the influence of adsorption force between gas molecules and rock surface is remarkable. When rock contains other polar fluid such as water or oil, the effect of gas TPG will be enhanced.

As seen in the second stage of Fig. 1, with the increase of gas pressure, gas molecule density increases, the interaction between gas molecules enhances, and the higher gas pressure, the lower the gas permeability. In this stage, the effect of interaction between gas molecules is remarkable.

As seen in the third stage of Fig. 1, slippage effect weakens gradually and gas enters the stage of stable Darcy flow, when the driving force is in the dominate status.

As seen in the fourth stage of Fig. 1, with more increase of air pressure, gas density becomes large, gas flow properties tend to be similar to the liquid flow, and gas flow is high velocity non-Darcy flow.

Due to the tiny pore throat, ultra-low permeability and high water saturation, the effect of TPG is the main influential factor of the production.

3. Conceptual model

3.1. Model assumption

Most hydraulic fracturing of horizontal wells are perforated after sealed with packer. In this case, fluid flow from matrix to wellbore is not considered.

Assumptions are given before establishing production forecast model of fractured horizontal as well as the reservoir is homogeneous and isotropic, gravity and capillary pressure are neglected, as well as wellbore storage and skin effect, fracture is symmetric around wellbore and located in the middle of layer with finite conductivity, fluid flows into fracture first and then flows into horizontal wellbore and fluid is only produced through perforation and fracture in horizontal well with case hole completion.

When the fractures of horizontal well are transverse, fluid flow could be divided into two parts: ellipse-shaped flow from stratum to cracks in horizontal plane, which is called external flow field, and linear and radial flow along cracks in vertical plane, which field is called internal flow field.

4. Mathematical model of fractured horizontal well

4.1. Productivity model of single fracture

4.1.1. Elliptical flow. According to Liu Ciqun, flow field of horizontal well could be recognized as isobaric ellipsoid and a family of hyperboloid streamline. Relationship between rectangular coordinate system and ellipsoid coordinate system is as follows:

$$x = a\cos\eta\,,\ y = b\sin\eta\,,$$

$$a = x_{\mathrm{f}}\cosh(\xi)\,,\ b = x_{\mathrm{f}}\sinh(\xi)\,, \tag{1}$$

where a and b are the major and minor axes of the ellipse, respectively, and x_{f} is the half-length of fracture.

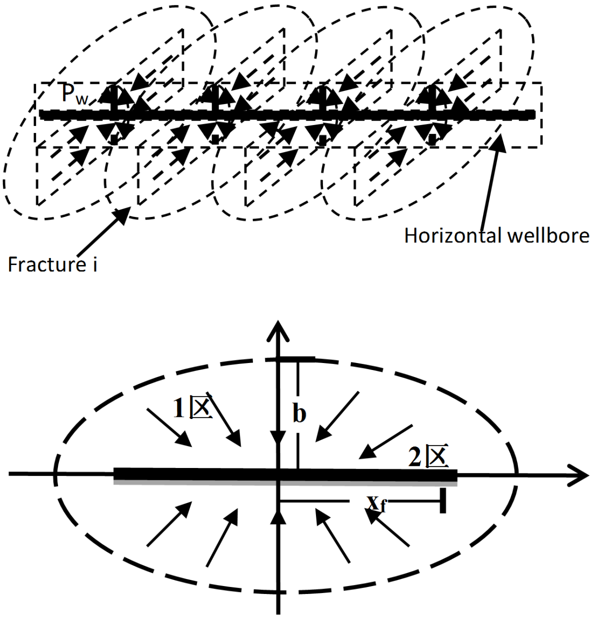

Fig. 2. Simplified schematic of flow field of fractured horizontal well

The mass conservation equation reads

$$\operatorname{div}\,(\rho_g \boldsymbol{v}) = 0\,, \tag{2}$$

where ρ_g is the density of gas and $\boldsymbol{v}$ is its velocity.

The gas state equation may be expressed in the form:

$$\rho_g = \frac{T_{sc} Z_{sc} \rho_{gsc}}{p_{sc}} \cdot \frac{p}{TZ}\,. \tag{3}$$

Here, T_{sc} is the standard state temperature, p_{sc} is the standard state pressure, Z_{sc} is the compressibility factor at the standard state and ρ_{gsc} is the density of gas at the standard state. Further, T, p and Z are the temperature, pressure and compressibility, respectively, in the real conditions.

The fluid flow follows the Darcy's law in elliptical flow region:

$$\boldsymbol{v} = \frac{k}{\mu}(\operatorname{grad}\, p - \boldsymbol{G})\,, \tag{4}$$

where k is the permeability of gas reservoir, μ is the gas viscosity, and $\boldsymbol{G}$ denotes the specific gravitational force acting on unit volume of gas.

Incorporating (3) and (4) into (2), and using a Cartesian coordinates to describe isobaric ellipse family, we get

$$\frac{\mathrm{d}\,(\rho v)}{\mathrm{d}y} = \frac{kT_{\mathrm{sc}}Z_{\mathrm{sc}}\rho_{\mathrm{gsc}}}{p_{\mathrm{sc}}T}\left[\frac{\partial}{\partial y}\left(\frac{p}{\mu Z}\frac{\partial p}{\partial y}\right) - C_\rho G\frac{p}{\mu Z}\frac{\partial p}{\partial y}\right] = 0\,, \tag{5}$$

where C_ρ is the gas compressibility coefficient.

Introducing the pseudo-pressure equation

$$m^* = 2\int_{p_{\mathrm{a}}}^{p} \frac{p}{\mu(p)Z(p)}\,\mathrm{d}p\,, \tag{6}$$

where p_{a} is a known pressure. Well-suited to engineering needs, $\mu(p)Z(p)$ can be simplified as μZ, the value of $\mu(p)Z(p)$ under constant temperature and average pressure of formation.

In combination with (5) and (6) we obtain the general governing equation in the form

$$\frac{\mathrm{d}^2 m^*}{\mathrm{d}y^2} - C_\rho G\frac{\mathrm{d}m^*}{\mathrm{d}y} = 0 \tag{7}$$

with constant-production inner boundary

$$\xi = \xi_{\mathrm{w}}\,, \quad y\frac{\mathrm{d}m^*}{\mathrm{d}y} = \frac{p_{\mathrm{sc}}Tq_{\mathrm{sc}}}{k\rho_{\mathrm{gsc}}T_{\mathrm{sc}}Z_{\mathrm{sc}}hx_{\mathrm{f}}\cosh(\xi)}\,. \tag{8}$$

The average minor axis of the inner boundary is

$$\bar{y}_{\mathrm{w}} = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} y\,\mathrm{d}\eta = \frac{2x_{\mathrm{f}}\sinh(\xi_{\mathrm{w}})}{\pi}\,. \tag{9}$$

The constant-pressure outer boundary is characterized by the conditions

$$\xi = \xi_{\mathrm{i}}\,, \quad p = p_{\mathrm{e}}\,, \quad m^* = m^*_{\mathrm{e}}\,, \tag{10}$$

Here, ξ_{w} is the elliptic coordinate near the wellbore, ξ_{i} is the elliptic coordinate of flow region and p_{e} is the initial reservoir pressure.

The average minor axis of the outer boundary is

$$\bar{y}_{\mathrm{i}} = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} y\,\mathrm{d}\eta = \frac{2x_{\mathrm{f}}\sinh(\xi_{\mathrm{i}})}{\pi}\,. \tag{11}$$

The analytical solution can be written in the form

$$-m_{\mathrm{xf}} + m_{\mathrm{e}} = \frac{p_{\mathrm{sc}}Tq_{\mathrm{sc}}}{2k\rho_{\mathrm{gsc}}T_{\mathrm{sc}}Z_{\mathrm{sc}}x_{\mathrm{f}}h\cosh(\xi_{\mathrm{w}})C_\rho G}\left[\mathrm{e}^{\frac{2x_{\mathrm{f}}C_\rho G}{\pi}(\sinh(\xi_{\mathrm{i}})-\sinh(\xi_{\mathrm{w}}))} - 1\right]\,. \tag{12}$$

4.2. High-velocity gas flow in fractures

The equation of motion in the fractures can be described as

$$-\mathrm{grad}\, p = \frac{\mu}{k_{\mathrm{f}}} v + \beta \rho v^2 \,, \tag{13}$$

where k_{f} is the permeability of fracture and β is the high-speed nonlinear flow coefficient. This coefficient can be determined from formula

$$\beta = \frac{0.005}{k_{\mathrm{f}}^{0.5} \varphi^{0.5}} \,. \tag{14}$$

The mass flow rate dividing cross-sectional area yields velocity

$$v = \frac{q_{\mathrm{m}}}{\rho_{\mathrm{g}} A} = \frac{\rho_{\mathrm{gsc}} q_{\mathrm{sc}}}{2\rho_{\mathrm{g}} w h} \,. \tag{15}$$

Integrating (12) from 0 to x_{f}, the analytical productivity equation in fracture obtains the form

$$m_{\mathrm{xf}} - m_{\mathrm{rw}} = \frac{2 p_{\mathrm{sc}} T x_{\mathrm{f}}}{k_{\mathrm{f}} T_{\mathrm{sc}} Z_{\mathrm{sc}} w h} q_{\mathrm{sc}} + \frac{2 p_{\mathrm{sc}} T \rho_{\mathrm{gsc}} x_{\mathrm{f}} \beta}{T_{\mathrm{sc}} Z_{\mathrm{sc}} w^2 h^2 \mu} q_{\mathrm{sc}}^2 \,. \tag{16}$$

4.3. Productivity equation in single fracture

Adding (11) and (16) yields productivity equation in one single fracture,

$$\begin{aligned} m_{\mathrm{e}} - m_{\mathrm{rw}} = {} & \frac{2 p_{\mathrm{sc}} T \rho_{\mathrm{gsc}} x_{\mathrm{f}} \beta}{T_{\mathrm{sc}} Z_{\mathrm{sc}} w^2 h^2 \mu} q_{\mathrm{sc}}^2 + \\ & + \left(\frac{p_{\mathrm{sc}} T}{2 k \rho_{\mathrm{gsc}} T_{\mathrm{sc}} Z_{\mathrm{sc}} x_{\mathrm{f}} h \cosh(\xi_{\mathrm{w}}) C_\rho G} \left[\mathrm{e}^{\frac{2 x_{\mathrm{f}} C_\rho G}{\pi} (\sinh(\xi_{\mathrm{i}}) - \sinh(\xi_{\mathrm{w}}))} - 1 \right] + \frac{2 p_{\mathrm{sc}} T x_{\mathrm{f}}}{k_{\mathrm{f}} T_{\mathrm{sc}} \mathrm{Z}_{\mathrm{sc}} w h} \right) q_{\mathrm{sc}} . \end{aligned} \tag{17}$$

As (17) is a quadratic polynomial, the productivity formula is derived as

$$q_{\mathrm{sc}} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \,, \tag{18}$$

where

$$a = \frac{2 p_{\mathrm{sc}} T \rho_{\mathrm{gsc}} x_{\mathrm{f}} \beta}{T_{\mathrm{sc}} Z_{\mathrm{sc}} w^2 h^2 \mu} \,,$$

$$b = \frac{p_{\mathrm{sc}} T}{2 k \rho_{\mathrm{gsc}} T_{\mathrm{sc}} Z_{\mathrm{sc}} x_{\mathrm{f}} h \cosh(\xi_{\mathrm{w}}) C_\rho G} \left[\mathrm{e}^{\frac{2 x_{\mathrm{f}} C_\rho G}{\pi} (\sinh(\xi_{\mathrm{i}}) - \sinh(\xi_{\mathrm{w}}))} - 1 \right] + \frac{2 p_{\mathrm{sc}} T x_{\mathrm{f}}}{k_{\mathrm{f}} T_{\mathrm{sc}} \mathrm{Z}_{\mathrm{sc}} w h} \,,$$

and

$$c = m_{\mathrm{rw}} - m_{\mathrm{e}} \,.$$

4.4. Productivity equation of fractured horizontal well

When the horizontal well contains hydraulically caused several fractures, and some flow regions formed by fractures have interference, it is possible to reduce them to the one region. Assuming that every two elliptical flow regions intersect, the intersection area is S_i, and the production of single fracture considering the interference is

$$q_{\mathrm{f}i} = \left(1 - \frac{S_i}{\pi a_i b_i}\right) q_{\mathrm{sc}}\,, \quad i = 1, \cdots, n\,, \tag{19}$$

where S_i is the ith intersection area, $q_{\mathrm{f}i}$ is the gas production of the ith single fracture and n is the number of fractures. The ith intersection area is

$$S_i = 2 \cdot \left(\frac{1}{4}\pi a_i b_i - \frac{1}{2} a_i b_i \arccos \frac{y_i}{b_i}\right) - \frac{W_i}{2} \cdot y_i + \\ + 2 \cdot \left(\frac{1}{4}\pi a_{i+1} b_{i+1} - \frac{1}{2} a_{i+1} b_{i+1} \arccos \frac{y_i}{b_{i+1}}\right) - \frac{W_{i+1}}{2} \cdot y_{i+1}\,, \tag{20}$$

where W_i is the width of the ith fracture ($i = 1, \cdots, n - 1$) and

$$x_i = \frac{W_i}{2}\,, \quad y_i = \sqrt{\left[1 - \left(\frac{W_i}{2a_i}\right)^2\right] \cdot b_i^2}\,, \quad i = 1, 2, \cdots, n - 1\,.$$

When there is no interference, $S_i = 0$.

The productivity prediction equation of fractured horizontal well in tight sandstone reservoir was presented considering the interference between fractures

$$Q_{\mathrm{sc}} = \sum_{i=1}^{n} q_{\mathrm{f}i}\,, \tag{21}$$

where Q_{sc} is the gas production of the horizontal well.

5. Analysis of influential factors

Numerical analysis is carried out using data of physical properties in Changling gas field. Its parameters are as follows: reservoir thickness is 8 m, initial formation pressure is 35 MPa, well bottom pressure is 25 MPa, TPG is 0.01 MPa·m^{-1}, gas viscosity at standard state is 0.027 mPa·s, gas density is 0.75 kg·m^{-3}, temperature in standard state is 293 K, reservoir temperature is 396 K, gas compressibility factor is 0.89, radius of wellbore is 0.1 m, porosity is 0.08, permeability is 0.1 mD, length of horizontal well is 1200 m, fracture quantity is 15, half-length of fracture is 150 m, fracture width is 3 mm, and permeability of fracture is 1 D.

Fig. 3 shows the relationship between production rate and pressure drawdown under different fracture half-length. As seen in the figure, production rate increases with the increase of pressure drawdown. Besides, production rate increases with increasing fracture half-length under certain drawdown, and half-length of fracture that ranges from 150 m to 200 m have a good effect to productivity.

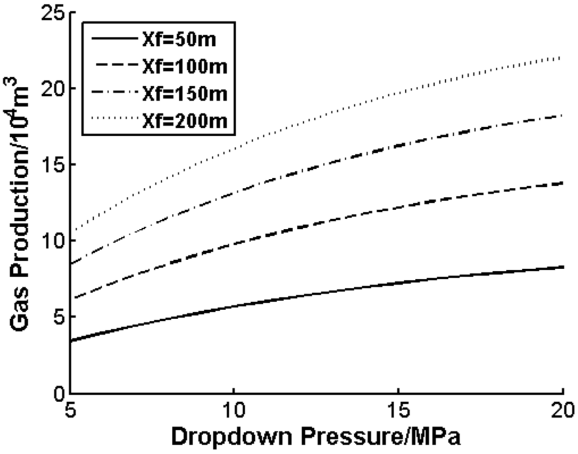

Fig. 3. Relationship between dropdown pressure and productivity under different half-lengths of fractures

Fig. 4 shows the relationship between production rate and pressure drawdown under different fracture conductivity. As seen in the figure, production rate increases with the increase of pressure drawdown. Besides, production rate increases with increasing fracture conductivity under certain drawdown. Improvement of the fracture conductivity have a good effect on productivity.

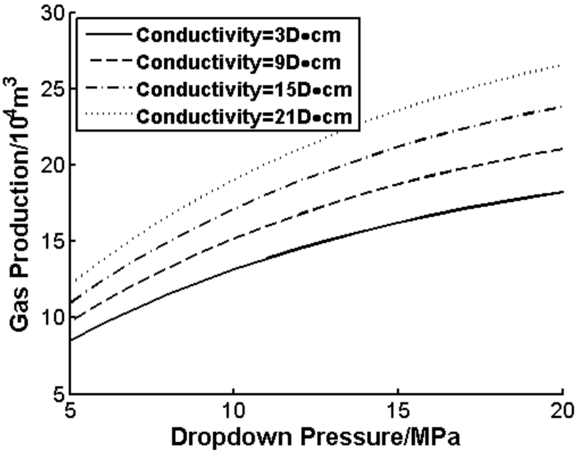

Fig. 4. Relationship between dropdown pressure and productivity under different conductivities of fractures

Fig. 5 shows the relationship between production rate and pressure drawdown under different fracture quantity. As seen in the figure, production rate increases with the increase of pressure drawdown under certain fracture quantity. Besides, production rate increases with increasing fracture quantity under certain drawdown. With increasing fracture quantity, production of fractured horizontal well increases with decreasing increment and approaches a plateau due to the interference.

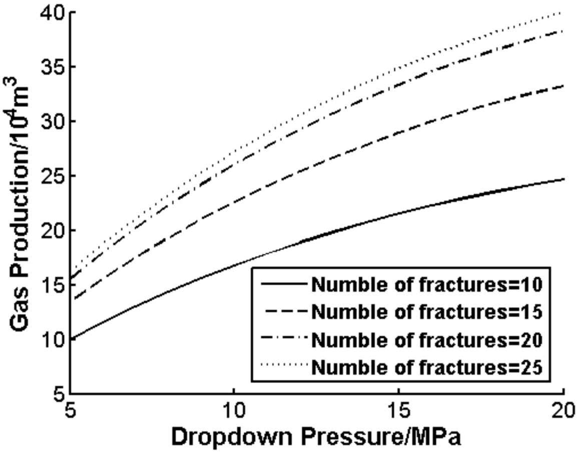

Fig. 5. Relationship between dropdown pressure and productivity under different fracture quantities

Fig. 6 shows the relationship between production rate and pressure drawdown under different TPG. As seen in the figure, production rate increases with the increase of pressure drawdown under certain TPG. Besides, production rate decreases with increasing TPG under certain drawdown. The fluid is hard to flow, and control area of well decreases due to the existence of TPG.

6. Conclusion

- The phenomena of non-Darcy flow in tight gas reservoir (TPG, slippage effect, and high-velocity non-Darcy flow) can be theoretically explained from molecular force, for high water-bearing tight gas reservoir, TPG being the main influential factor.

- According to different flow regions around the fractured horizontal well, considering the existence of threshold pressure gradient when the fluid flow in tight water-bearing gas reservoir, a mathematical model about fractured horizontal well coupled with two flow regions was established ,as the pressure at the junction of two regions equal, productivity equation of single fracture could be presented.

- The longer the fracture half-length, the bigger the productivity, but the productivity growth is reduced, and there is an optimal fracture half-length; the

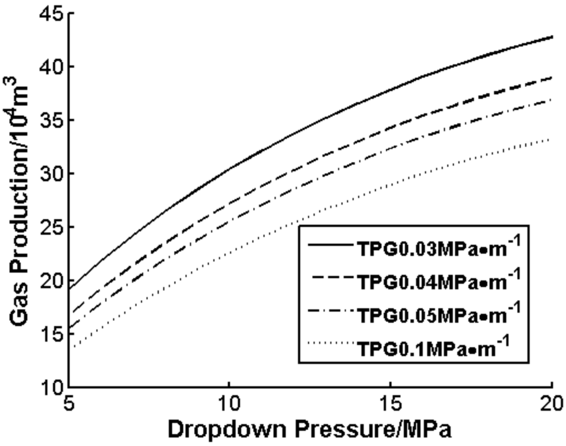

Fig. 6. Relationship between dropdown pressure and productivity under different values of TPG

bigger the fracture conductivity, the bigger the productivity; the more the fracture quantity, the bigger productivity, but productivity growth is reduced due to the interference between fractures.

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Received November 16, 2016